

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY- GURAJADA VIZIANAGARAM**  
**II B. Tech I Semester Supplementary Examinations, November – 2024**  
**RANDOM VARIABLES AND STOCHASTIC PROCESSES**  
**(ECE)**

Time: 3 hours

Max. Marks: 70

*Answer any FIVE Questions*  
*ONE Question from Each unit*  
*All Questions Carry Equal Marks*  
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- 1 a) Write short notes below [7]
    - i. Binomial random variable
    - ii. Poisson Random variable
  - b) A random voltage can have any value defined by the set 'S' = {a ≤ s ≤ b}. A quantizer, divides S into 6 equal-sized contiguous subsets and generates random variable X having values {-4, -2, 0, 2, 4, 6}. Each value of X is earned to the midpoint of the subset of 'S' from which it is mapped i) Sketch the sample space and the mapping to the line that defines the values of X ii) Find a and b? [7]
- (OR)
- 2 a) Define Random variable? List out the properties of Distribution Function [7]
  - b) Suppose height to the bottom of clouds is a Gaussian random variable for which  $\mu_x = 4000\text{m}$  and  $\sigma_x = 1000\text{m}$ . A person bets that cloud height tomorrow will fall in the set  $A = \{1000\text{m} < X \leq 3000\text{m}\}$  while a second person bets that height will be satisfied by  $B = \{2000\text{m} < X \leq 4200\text{m}\}$ . A third person bets they are both correct. Find the probability that each person will win the bet. [7]
- 3 a) Discuss in detail about Functions that gives Moments? [7]
  - b) Find the mean, variance from moment generation function of uniform distribution? [7]
- (OR)
- 4 a) A random variable X is uniformly distributed on the interval  $(-\pi, \pi)$ . X is transformed to the new random variable  $Y = T(x) = a \tan(X)$ , where  $a > 0$ . Find the probability density function of [7]
  - b) State and prove Chebychev's inequality. [7]
- 5 a) define random variables V and W by [7]
 
$$V = X + aY$$

$$W = X - aY$$

Where a is real number and X and Y random variables, Determine a in terms of X and Y such V and W are orthogonal?
  - b) Two Gaussian random variables X and Y are variances  $\sigma_X^2 = 9$  and  $\sigma_Y^2 = 4$  respectively and correlation coefficient  $\rho$ . It is known that a coordinate rotation by angle  $-\pi/8$  results in new random variable  $Y_1$  and  $Y_2$  that are uncorrelated. [7]
- (OR)
- 6 a) Random variables X and Y have respective density functions [9]
 
$$f_X(x) = \frac{1}{a}[u(x) - u(x-a)]$$

$$f_Y(y) = \frac{1}{b}[u(y) - u(y-b)]$$

Where  $b > a$  and  $a > 0$ . Find and sketch the density functions of  $W = X + Y$  if X and Y are statistically independent

- b) Three statistical independent Random variables  $X_1, X_2, X_3$  are defined by [5]

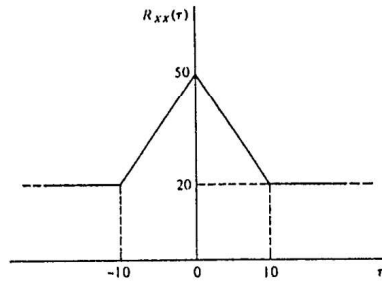
$$\begin{aligned}\bar{X}_1 &= -1 & \sigma_{X_1}^2 &= 2.0 \\ \bar{X}_2 &= 0.6 & \sigma_{X_2}^2 &= 1.5 \\ \bar{X}_3 &= 1.8 & \sigma_{X_3}^2 &= 0.8\end{aligned}$$

Write the equation describing the Gaussian approximation for the density function of the sum  $X = X_1 + X_2 + X_3$ .

- 7 a) Write the properties of Autocorrelation Function of Random Process [7]  
 b) Give the random process by  $X(t) = A \cos(w_0 t) + B \sin(w_0 t)$  [7]  
 Where  $w_0$  is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance, show that X(t) is wide sense stationary but not strictly stationary

(OR)

- 8 a) For stationary ergodic random process having the autocorrelation function shown in below figure then find the [7]  
 i. Mean  
 ii. variance



- b) Given that the autocorrelation function for a stationary Ergodic process with no period components is [7]

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of process X(t)?

- 9 a) Derive the relationship between power spectrum and auto-correlation [7]  
 b) If X(t) is a stationary process, find the power spectrum of  $Y(t) = A_0 + B_0 X(t)$  in term [7]  
 of the power spectrum of X(t) if  $A_0$  and  $B_0$  are real constants

(OR)

- 10 A Random signal X(t) of PSD of  $\frac{N_0}{2}$  is applied on an LTI system having [14]  
 impulse response  $h(t)$ . If Y(t) is output, find (i)  $E[Y^2(t)]$  (ii)  $R_{XY}(\tau)$  (iii)  $R_{YX}(\tau)$   
 (iv)  $R_{YY}(\tau)$

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